Meander-Folded Coupled Lines

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Abstract—**The quarter wave meander-folded coupler is a compact device useful for frequencies up to a few gigahertz. The folding is shown to shorten the electric length of the coupler and a theory is presented for the calculation of this shortening effect. The theory shows that the even-mode circuit is shortened to a higher extent than the odd-mode circuit, which in stripline causes a degradation of performance. A conformal mapping technique is presented for determining the characteristic impedances of some stripline structures necessary for the theory.**

The theory is verified by experimental meander-folded stripline couplers. It is also experimentally shown that the meander-folded microstrip coupler can be designed to equalize the even- and the odd-mode lengths.

I. INTRODUCTION

THERE ARE SEVERAL applications of microwave components where size and weight have to be minimized. For frequencies up to a few gigahertz the method of folding coupled transmission lines in a meander pattern is useful. An application using meander-folded coupled lines was presented recently [1], [2], but only a qualitative theory was given. It was shown that the folding of a coupled line in a meander pattern shortens the even- and the odd-mode electric lengths. Since the even mode is shortened to a higher extent than the odd mode, this causes a degradation of performance. By equalizing the mode lengths the performance of the meander-folded coupler can be improved.

The microstrip coupler suffers from a similar effect. Since the even-mode field lies more in the dielectric than the odd-mode field, the even-mode circuit is electrically longer. A number of methods to equalize these lengths have been presented [3]; one is known as the wiggly coupler.

The theory of the meander-folded coupler presented in Section II makes it possible to calculate the shortening of the even- and the odd-mode lengths. Some stripline impedance calculations required for the theory are derived in Section III. Experimental results are presented in Section IV and they are compared with the theoretical ones. It is also verified that the folded microstrip coupler can be designed to equalize the even- and the odd-mode lengths.

II. THEORY OF THE MEANDER-FOLDED COUPLER

The pattern of a meander-folded coupler is shown in Fig. 1. The unfolded line is a quarter-wave at the design frequency. If the two lines are considered as a unit we see a meander line with the distance d between its branches.

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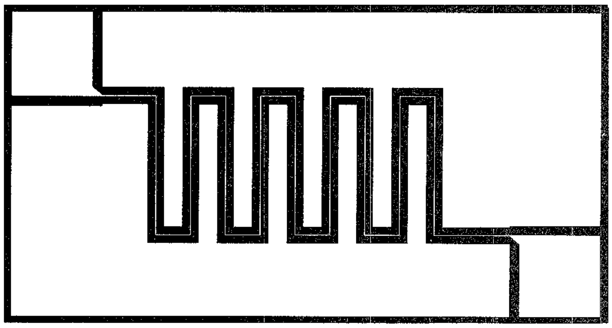

Fig. 1. The pattern of a meander-folded coupler.

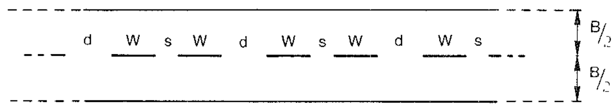

Fig. 2. Cross section of coupled lines in the stripline case.

A straight quarter-wave line becomes electrically shorter when it is folded into a meander pattern [4]. A more detailed analysis of the folded coupler also shows this shortening effect. An even- and odd-mode analysis shows that the even-mode circuit is shortened to a higher extent than the odd-mode circuit. Different lengths of the two modes cause a degradation of VSWR and isolation of the coupler.

Suppose that the double meander pattern consists of several branches and that the coupling between nonadjacent lines is low. We also assume that d is small enough so that the length of the lines connecting the meander branches can be neglected. It is then possible to analyze the pattern in Fig. 1 as a part of an infinite array of coupled lines with the symmetric cross section shown in Fig. 2.

The condition of low coupling between nonadjacent lines allows us to split the cross section in Fig. 2 with a number of even and odd modes [1], [5]. Fig. 3 shows the different modes involved in such an analysis.

There are four different characteristic impedances we have to know in Fig. 3. The figure shows the physical configurations corresponding to ZE_e, ZE_o, ZO_e, and ZO_o. The only difference between the structures is the nature of the sidewalls which can be electric and/or magnetic. Although it may not be necessary for the theory presented, d is expected to be larger than s; that is, the coupling between the two lines of the coupler is tighter than the coupling across the distance d. The unfolded coupler is characterized by the even-mode impedance

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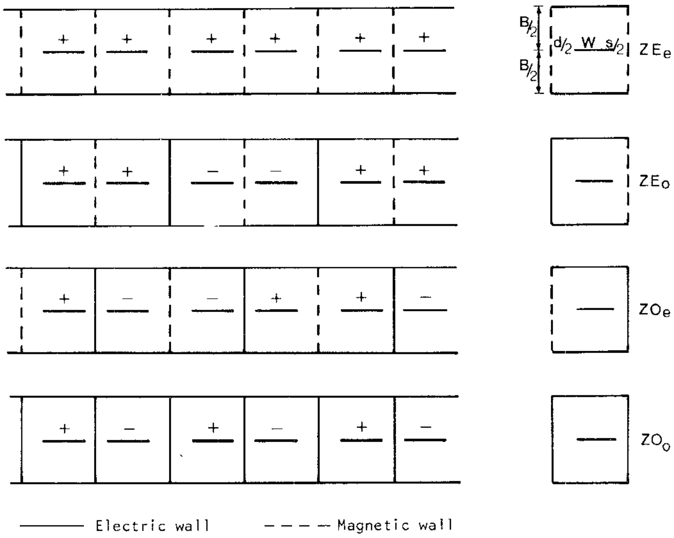
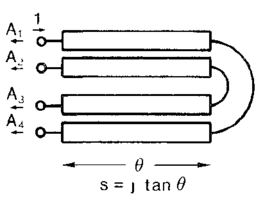


Fig. 3. Even and odd modes for analyzing an infinite array of coupled lines.



$$A_1 = \frac{\Gamma E_e + \Gamma E_o + \Gamma O_e + \Gamma O_o}{4}$$

$$A_2 = \frac{\Gamma E_e + \Gamma E_o - \Gamma O_e - \Gamma O_o}{4}$$

$$A_3 = \frac{\Gamma E_e - \Gamma E_o - \Gamma O_e + \Gamma O_o}{4}$$

$$A_4 = \frac{\Gamma E_e - \Gamma E_o - \Gamma O_e + \Gamma O_o}{4}$$

Circuit	Inimpedance	Reflection coefficient
	$\frac{ZE_e}{s}$	$\Gamma E_e = \frac{ZE_e - s}{ZE_e + s}$
	sZE_o	$\Gamma E_o = \frac{sZE_o - 1}{sZE_o + 1}$
	$\frac{ZO_e}{s}$	$\Gamma O_e = \frac{ZO_e - s}{ZO_e + s}$
	sZO_o	$\Gamma O_o = \frac{sZO_o - 1}{sZO_o + 1}$

Fig. 4. Analyzing a section of the meander-folded coupler with the modes in Fig. 3.

$ZE = ZE_e = ZE_o$ and the odd-mode impedance $ZO = ZO_e = ZO_o$ which may explain the selected notations.

Formulas for calculating these impedances in stripline are given in Section III. The calculations are based on conformal mapping and constitute an extension of earlier work [6].

Fig. 4 shows how a section of the meander-folded coupler can be analyzed using the modes in Fig. 3. Several equal sections can then be cascaded until the desired number of branches is achieved. The number of branches must be even for this method of analysis. We note that Θ is not 90° at the design frequency, since it is only a section of the total coupler that is analyzed in Fig. 4.

We define some new parameters

$$ZE = \sqrt{ZE_e \cdot ZE_o} \tag{1a}$$

$$ZO = \sqrt{ZO_e \cdot ZO_o} \tag{1b}$$

$$ZC = \sqrt{ZE \cdot ZO} \tag{1c}$$

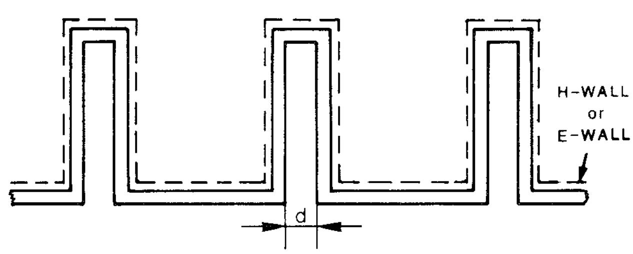


Fig. 5. Cascaded Schiffman sections.

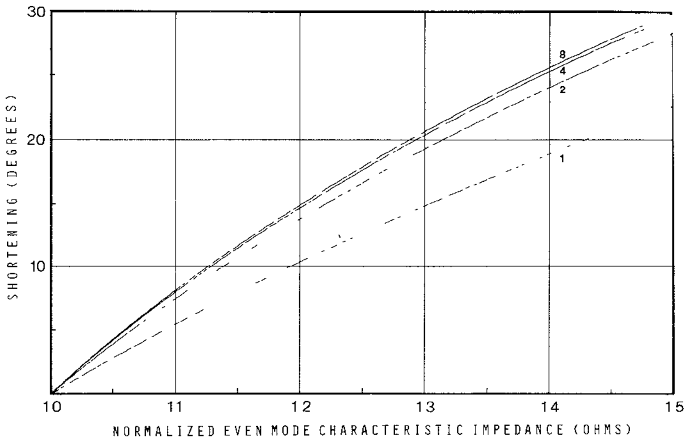


Fig. 6. Shortening of 1, 2, 4 resp. 8 cascaded Schiffman sections. The insertion phase of the unfolded line is 90°.

$$Z_e N = \sqrt{\frac{ZE_e}{ZE_o}} = \frac{ZE_e}{ZE} \tag{1d}$$

$$Z_o N = \sqrt{\frac{ZO_e}{ZO_o}} = \frac{ZO_e}{ZO}. \tag{1e}$$

The conditions for perfect match and isolation have been calculated.

$$A_1 = A_4 = 0$$

when

$$Z_e N = Z_o N$$

$$ZC = 1, \qquad 1\text{-}\Omega \text{ terminations.} \tag{2}$$

An alternate method to the one in Fig. 4 has been used. If the magnetic or the electric wall is inserted between the two lines of the coupler, the pattern in Fig. 1 reduces to that in Fig. 5. The simplified pattern consists of a number of cascaded Schiffman sections.

Since the total physical length of the folded line is a quarter-wave at the design frequency, the length of each Schiffman section is much shorter. When a transmission line shorter than a half-wave is folded into a Schiffman section, its electric length becomes shorter due to the coupling between the lines [1]. In Fig. 6 this shortening is seen as a function of the normalized even-mode characteristic impedance of each cascaded Schiffman section. The unfolded line is a quarter-wave, i.e., its electric length is 90°.

As explained in [2] there will be a difference whether the dashed line in Fig. 5 represents an electric or magnetic wall. In the odd-mode case, when an electric wall is

TABLE I
CALCULATED IMPEDANCES AND SHORTENING OF MEANDER-FOLDED STRIPLINE COUPLERS; THE INSERTION PHASE OF THE UNFOLDED COUPLER IS 90°

ϵ	B (mm)	S (mm)	W (mm)	d (mm)	ZE_e (Ω)	ZE_o (Ω)	ZO_e (Ω)	ZO_o (Ω)	ZE (Ω)	ZO (Ω)	ZC (Ω)	Z_eN (Ω)	Z_oN (Ω)	Shortening: Even mode (degrees)	Shortening: Odd mode (degrees)	Shortening: Difference (degrees)
2.62	3.18	0.16	1.86	4.	70.56	69.54	35.79	35.53	70.05	35.66	49.98	1.01	1.00	0.6	0.3	0.3
				3.	71.42	68.67	36.00	35.31	70.03	35.65	49.97	1.02	1.01	1.8	0.9	0.9
				2.	73.71	66.28	36.54	34.70	69.90	35.61	49.89	1.05	1.03	4.7	2.3	2.4
				1.	79.70	59.47	37.90	32.81	68.85	35.26	49.27	1.16	1.07	12.3	6.3	6.0
				0.5	85.68	51.50	39.16	30.30	66.42	34.45	47.83	1.29	1.14	20.2	10.8	9.4
		0.16	1.70	4.	75.16	74.00	37.00	36.72	74.58	36.86	52.43	1.01	1.00	0.7	0.3	0.4
				3.	76.14	73.01	37.21	36.49	74.56	36.85	52.42	1.02	1.01	1.9	0.9	1.0
				2.	78.74	70.32	37.79	35.85	74.41	36.81	52.34	1.06	1.03	5.0	2.3	2.6
				1.	85.62	62.70	39.22	33.86	73.27	36.44	51.67	1.17	1.08	13.0	6.4	6.6
				0.5	92.55	53.91	40.55	31.22	70.64	35.58	50.13	1.31	1.14	21.3	11.0	10.3

inserted, the field will be closely confined to the electric wall and the coupling across the distance d will have a minor effect. As seen in Section III this is not the case when the magnetic wall is inserted. The result is that the even mode of the meander-folded coupler is shortened more than the odd mode, which degrades the performance.

Table I shows some calculations on meander-folded couplers using the "cascaded Schiffman section" theory. It is seen in the table that the conditions for a perfect match (2) cannot be fulfilled. The shortening has been calculated using the normalized even-mode impedance of the Schiffman sections. With a magnetic wall in Fig. 5 this impedance is Z_eN, and with an electric wall the impedance is Z_oN. In stripline the difference in length between the two modes causes a degradation of the directivity of the coupler.

III. STRIPLINE IMPEDANCE CALCULATIONS

A method to calculate the characteristic impedance of the structures shown in Fig. 7 is presented in this section. Each structure consists of a strip symmetrically placed between two ground planes in a homogeneous dielectric. The structure in Fig. 7(a) consists of a single strip in a box. Two edge-coupled lines that are equal and symmetrically placed in a box are characterized by an even mode (Fig. 7(b)) and an odd mode (Fig. 7(a)) [1], [5]. The approximate theory of the meander-folded coupler presented in Section II requires the characteristic impedance of the structure in Fig. 7(c) as well as the other two in the same figure.

The physical dimensions are defined in Fig. 8. Since the strip is symmetrically placed between the ground planes, a magnetic wall can be inserted at the plane of symmetry.

Each stripline configuration in Fig. 7 can propagate a TEM mode. The characteristic impedance of a single TEM line is determined by the capacitance between the line and ground. For a strip with zero thickness the method of conformal mapping is useful. Ekinge [6] has presented conformal mapping functions for impedance calculations of the structures in Fig. 7(a) and Fig. 7(b). His method has been extended here to incorporate the structure in Fig. 7(c) as well. The symmetric case ($D = w + 2s$) has been treated before [7].

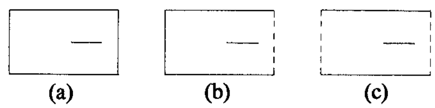


Fig. 7. Stripline configurations. (a) Electric sidewalls. (b) One electric and one magnetic sidewall. (c) Magnetic sidewalls.

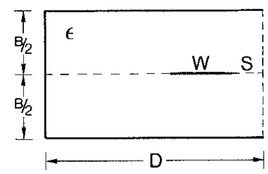


Fig. 8. Definition of stripline parameters.

Fig. 9 shows the mappings involved in the three different cases. As the figure shows, three steps are required to solve each configuration in Fig. 7. By using analytic mapping functions the mappings will be conformal, which implies that the capacitance between the conductors will be equal in all four planes. The configuration in the w_2-plane was selected so that the capacitance between the conductors would be easy to calculate.

The mapping function $f_1(z)$ as well as $f_2(z^1)$ in Fig. 9 are given by the Schwarz–Christoffel transformation formula. Since the mapping functions used are analytic in the upper half z-plane resp. z^1-plane the mappings are conformal. We obtain

$$f_1(z) = c_1 \int_0^z \left[(\zeta+1)\zeta(\zeta-\rho) \right]^{-1/2} d\zeta + c_2 \tag{3a}$$

$$f_2(z^1) = c_3 \int_0^{z^1} \left[(\eta+1)\eta(\eta+q) \right]^{-1/2} d\eta + c_4. \tag{3b}$$

The physical dimensions defined in Fig. 8 are used in the w_1-plane. With $f_1(-1) - f_1(-x_0) = s$ etc., and (3a) and (3b) (a detailed derivation is found in [6]), we find

$$\frac{2D}{B} = \frac{K(k_d)}{K(k_d^1)} \tag{4a}$$

$$\frac{2s}{B} = \frac{F\left(\arccos (x_0)^{-1/2}, k_d\right)}{K(k_d^1)} \tag{4b}$$

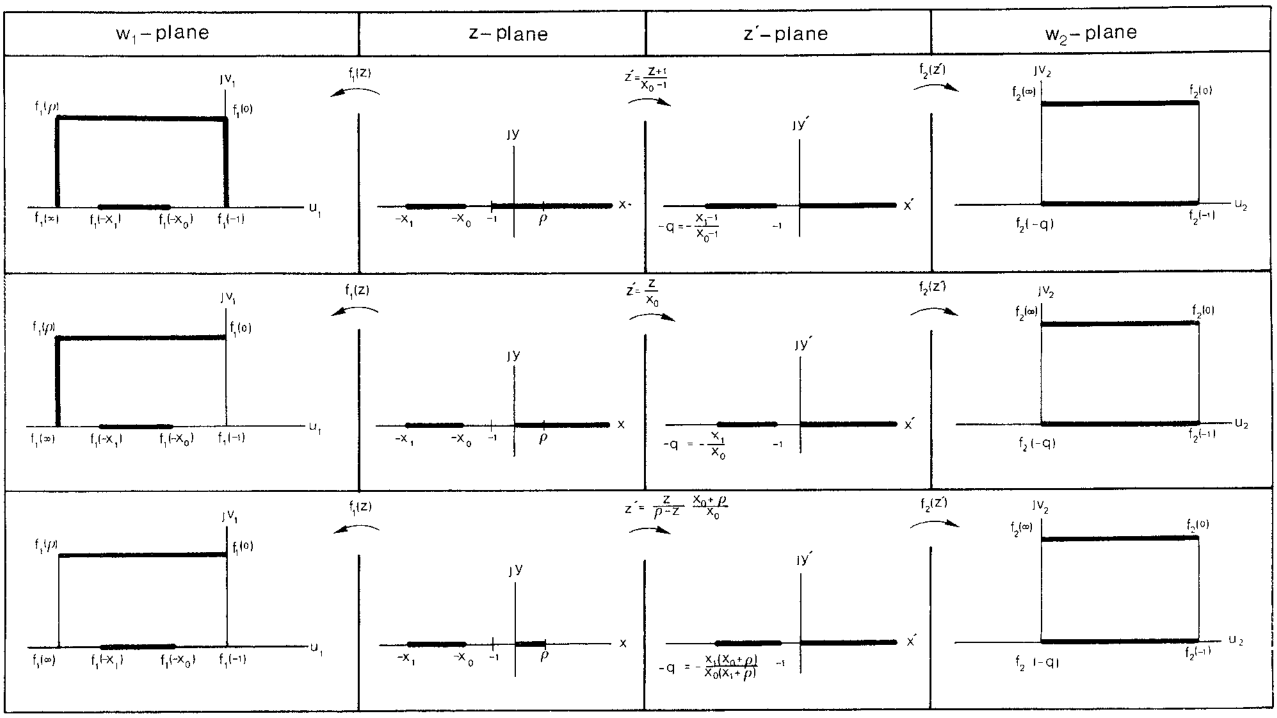


Fig. 9. Conformal mappings for capacitance calculation.

$$\frac{2w}{B}+\frac{2s}{B}=\frac{F\left(\arccos (x_1)^{-1/2}, k_d\right)}{K\left(k_d^1\right)} \tag{4c}$$

$$C=\epsilon_r\epsilon_0\frac{K\left(k_q^1\right)}{K\left(k_q\right)}, \qquad F/\text{unit length} \tag{4d}$$

where C is the capacitance per unit length between the strip and ground

$$F(\phi,k)=\int_0^{\sin\phi}\left[(1-\zeta^2)(1-k^2\zeta^2)\right]^{-1/2}d\zeta \tag{4e}$$

$F(\phi,k)$ is the Legendre normal integral of the first kind

$$K(k)=F\left(\frac{\pi}{2},k\right) \tag{4f}$$

$K(k)$ is the complete elliptic normal integral

$$k_d=\sqrt{\frac{\rho}{\rho+1}} \tag{4g}$$

$$k_d^1=\sqrt{1-k_d^2} \tag{4h}$$

$$k_q=\sqrt{\frac{1}{q}} \tag{4i}$$

$$k_q^1=\sqrt{1-k_q^2} \tag{4j}$$

B, D, s, w, x_0, x_1, q, and ρ are defined in Fig. 8 and Fig. 9.

When D/B is known, k_d is found by a zero searching routine. To calculate C it is necessary to know q which is a function of x_0, x_1, and ρ. x_0 and x_1 can be expressed explicitly in terms of the Jacobian elliptic function cn which is related to $F(\phi,k)$ through

$$\text{cn}\left(F(\phi,k),k\right)=\cos\phi. \tag{5}$$

We find

$$x_0=\frac{1}{\cos^2\phi_0} \tag{6a}$$

$$\cos\phi_0=\text{cn}\left[K\left(k_d^1\right)\frac{2s}{B},k_d\right] \tag{6b}$$

$$x_1=\frac{1}{\cos^2\phi_1} \tag{6c}$$

$$\cos\phi_1=\text{cn}\left[K\left(k_d^1\right)\frac{2(s+w)}{B},k_d\right]. \tag{6d}$$

The characteristic impedances of the configurations in Fig. 7 are obtained by

$$Z_c=\frac{\epsilon_r\epsilon_0 Z_0}{2C}=\frac{Z_0}{2}\frac{K\left(k_q\right)}{K\left(k_q^1\right)} \tag{7a}$$

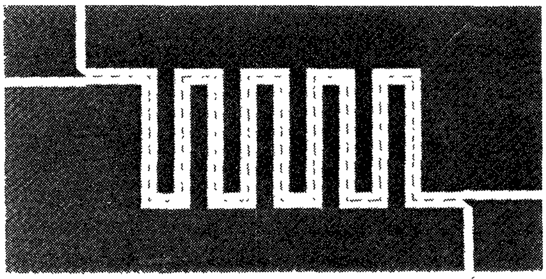
where

$$Z_0=\sqrt{\frac{\mu_0}{\epsilon_r\epsilon_0}}=\frac{1}{\sqrt{\epsilon_r}}\sqrt{\frac{\mu_0}{\epsilon_0}}. \tag{7b}$$

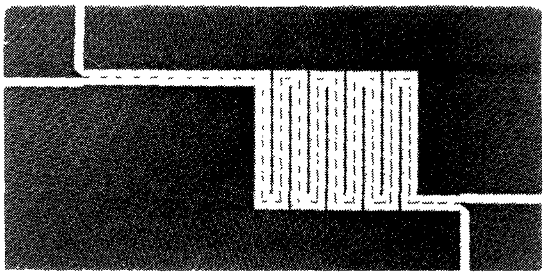
IV. Experimental Verification

A. Stripline in Homogeneous Dielectric

Experimental results for meander-folded couplers in homogeneous dielectric were presented recently [1], [2].



(a)



(b)

Fig. 10. Experimental meander-folded stripline couplers. (a) $d=4$ mm. (b) $d=0.5$ mm.

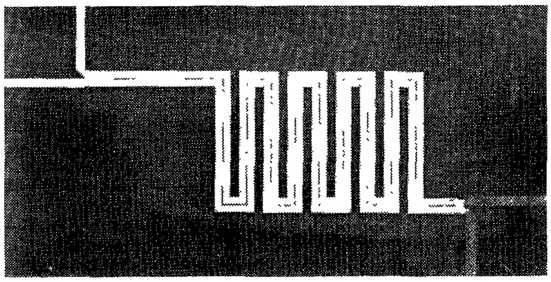


Fig. 11. Experimental meander-folded microstrip Schiffman section.

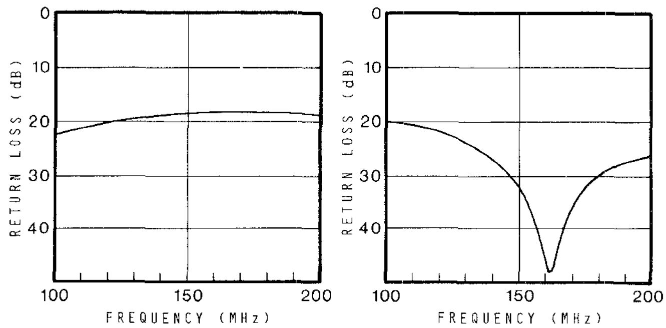


Fig. 12. Return loss of experimental microstrip Schiffman sections. (a) Without meander folding. (b) With meander folding as Fig. 11 shows.

For further verification of the theory presented here a number of experimental couplers have been examined. Two of the experimental stripline couplers are shown in Fig. 10. The two couplers have the same physical length, which is a quarter-wave at 130 MHz. The width of the strip w is 1.86 mm and $d=4$ resp. 0.5 mm. The other parameters are found in Table I.

When we calculate the shortening in Section II we assume that each connection between the cascaded Schiffman sections in Fig. 5 has zero length. This is not the case for the couplers in Fig. 10. For each coupler the physical length of the horizontal parts in the figure constitutes 25 percent of the total physical length. For this reason the theoretical shortening has to be reduced by 25 percent. For the coupler with $d=4$ mm, the shortening is less than 1° and can hardly be measured. When $d=0.5$ mm we obtain at the design frequency: even-mode shortening is 15.2° calculated and 14.9° measured. Odd-mode shortening is 8.1° calculated and 8.2° measured. Thus the agreement between calculated and measured shortening is very good.

B. Microstrip

To show that the even- and the odd-mode lengths of a microstrip coupler can be equalized by meander folding, a Schiffman section was manufactured. The reason for making a Schiffman section instead of a coupler is that the Schiffman section is more sensitive to a difference in the mode lengths, i.e., if the coupled section is well matched as a Schiffman section, it will be even better matched as a coupler. This was also verified by the experiments. The meander-folded Schiffman section is shown in Fig. 11. The dielectric constant is 2.62, the board thickness is 0.79 mm, $s=0.16$ mm, $w=1.76$ mm, and $d=2.00$ mm. The design frequency is 150 MHz.

The return loss of the circuit in Fig. 11 is compared to that of an experimental unfolded microstrip Schiffman section in Fig. 12. The unfolded Schiffman section is mismatched due to the different lengths of the even and the odd mode in a microstrip coupler. Here a low dielectric constant is used, hence the mismatch is moderate. For a dielectric constant of 10 (alumina substrate) the mismatch would be much worse.

We see that the meander folding can improve the match of a microstrip Schiffman section. As expected, the frequency for a perfect match is higher than the design frequency due to the shortening effect. It must be pointed out that the performance of the meander-folded device may be improved even further by optimizing the corners and the distance d.

C. Stripline in Nonhomogeneous Dielectric

The most recent experimental results were obtained at Anaren Microwave, Inc. Stripline couplers with broadside-coupled strips were meander folded to minimize the size of the device. To compensate for the shortening effect of the meander folding, a ground plane board with a dielectric constant of 4.0 is used together with a center board that has a dielectric constant of 2.6. The meander-folded coupler is shown in Fig. 13. The ground plane spacing is 1.79 mm, the center board thickness is 0.09 mm, $w=0.33$ mm, and $d=1.14$ mm. The design frequency is 450 MHz with an octave bandwidth and 3-dB coupling. The size of the meander pattern of the device in Fig. 13 is 14×12 mm^2.

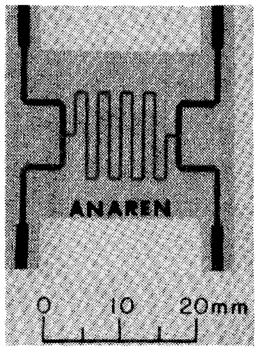


Fig. 13. Experimental meander-folded stripline coupler in a nonhomogeneous dielectric.

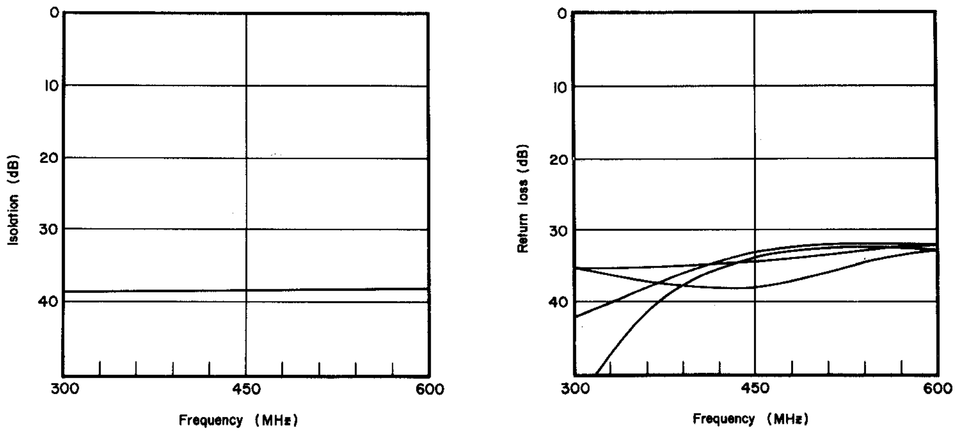


Fig. 14. Experimentally obtained data for the circuit in Fig. 13. (a) Isolation. (b) Return loss.

The experimentally obtained isolation and return loss are shown in Fig. 14. The figure shows that the isolation is better than 38 dB (⇒directivity > 34 dB) and the return loss is better than 32 dB. This circuit clearly shows the usefulness of meander folding. The optimum circuit dimensions were found with the help of a measurement method developed at Anaren [8]. The new method gives the even- and odd-mode impedances of a coupler and also the even- and odd-mode lengths.

V. Conclusions

The meander-folded coupler has been investigated theoretically and experimentally. The investigation shows that the folding causes a shortening of the even- and the odd-mode electric lengths. The shortening was found to be larger for the even- than for the odd-mode circuit. In stripline couplers the length difference between the two modes will cause a degradation of the directivity and the match. It has been shown experimentally that the presented theory is useful for calculating the shortening of the two modes.

A difficulty with microstrip couplers and stripline couplers in an inhomogeneous dielectric when the center board has a lower dielectric constant than the outer boards, is that the even-mode circuit will be electrically longer than the odd-mode circuit. This unwanted effect can be removed in the meander-folded coupler, which was verified by the experiments.

The next step would be to realize the meander-folded microstrip coupler for higher frequencies and on an alumina substrate. In this case there will be few meander branches and a new theory may be necessary.

Acknowledgment

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Parallel Line Microstrip Filters in an Inhomogeneous Medium

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***Abstract*—Parallel coupled microstrip lines in an inhomogeneous medium are studied. The quasi-static capacitance is shown to be linear with regard to the dielectric constant ϵ_r, simplifying the formalism used for analyzing microstrip filters.**

The electromagnetic advantages of the homogeneous medium carry over to the inhomogeneous medium. This result is obtained by equalizing all the velocities of the propagation modes.

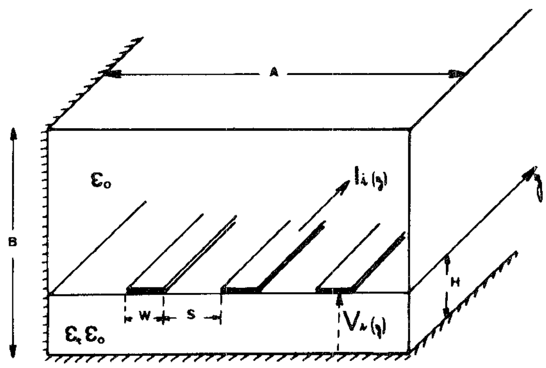

Fig. 1. Adjacent microstrip lines.

I. Introduction

A NUMBER OF interesting properties have been found recently in the course of a study of microstrip filters. Some of these properties facilitate study of these filters; others should lead to new applications.

Microstrip filters in an inhomogeneous medium have not been studied as fully as stripline filters in homogeneous medium, in spite of important basic papers devoted to this subject [1]–[6]. The design of microstrip filters at frequencies above 1 GHz should take into account the existence of hybrid modes [7]–[9]. In order to reduce the problem at first, we made the usual assumption of a quasi-TEM mode. Higher order modes are introduced later, after the filter design using the quasi-TEM mode [7]–[9], [13].

II. Summary of the New Results

Many devices previously studied [3] have a set of N parallel coupled propagating lines. In microstrip technology, we are interested in a set of parallel microstrips deposited on a dielectric substrate. The metallized lower dielectric surface is grounded. The whole system is shielded by a conducting box (Fig. 1). Various filter configurations are shown in Fig. 2. Our approach [10]–[12] may be summarized as a) an extension of Kirchhoff's theory to a system of N parallel coupled transmission lines, and b) the introduction of boundary conditions, reducing the system to a two-port filter, whose response will be calculated.

The new results may be summarized as follows.

1) Some parameters are perfectly linear with regard to the substrate ϵ_r. This point should allow easier studies for various values of ϵ_r.

2) A property of the various propagating modes allows the application of a new and simple formalism, making it possible to study filtering structures having a substantial number of lines coupled together.

3) The parallel microstrip structures in an inhomogeneous medium behave like structures in a homogeneous medium. The advantages of the homogeneous medium (such as width and regularity of the band) are maintained,

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